

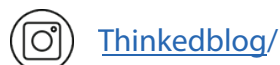
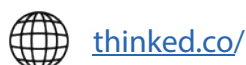


CSS Past Paper

# Pure Mathematics

(2017)

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**FEDERAL PUBLIC SERVICE COMMISSION  
COMPETITIVE EXAMINATION-2017  
FOR RECRUITMENT TO POSTS IN BS-17  
UNDER THE FEDERAL GOVERNMENT**

Roll Number

**PURE MATHEMATICS**

**TIME ALLOWED: THREE HOURS**

**MAXIMUM MARKS = 100**

- NOTE: (i)** Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C**. **ALL** questions carry **EQUAL** marks.
- (ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places.
- (iii)** Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- (iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- (v)** Extra attempt of any question or any part of the attempted question will not be considered.
- (vi) Use of Calculator is allowed.**

**SECTION-A**

- Q. 1. (a)** Let  $H, K$  be subgroups of a group  $G$ . Prove that  $HK$  is a subgroup of  $G$  if and only if  $HK=KH$ . (10)
- (b)** If  $N, M$  are normal subgroups of a group  $G$ , prove that (10) **(20)**  

$$NM/M \cong N/N \cap M.$$
- Q. 2. (a)** If  $R$  is a commutative ring with unit element and  $M$  is an ideal of  $R$  then show that  $M$  is a maximal ideal of  $R$  if and only if  $R/M$  is a field. (10)
- (b)** If  $F$  is a finite field and  $\alpha \neq 0, \beta \neq 0$  are two elements of  $F$  then show that we can find elements  $a$  and  $b$  in  $F$  such that (10) **(20)**  

$$1 + \alpha a^2 + \beta b^2 = 0.$$
- Q. 3. (a)** Let  $V$  be a finite-dimensional vector space over a field  $F$  and  $W$  be a subspace of  $V$ . Then show that  $W$  is finite-dimensional, (10)  

$$\dim W \leq \dim V \text{ and } \dim V/W = \dim V - \dim W.$$
- (b)** Suppose  $V$  is a finite-dimensional vector space over a field  $F$ . Prove that a linear transformation  $T \in A(V)$  is invertible if and only if the constant term of the minimal polynomial for  $T$  is not 0. (10) **(20)**

**SECTION-B**

- Q. 4. (a)** Use the Mean-Value Theorem to show that if  $f$  is differentiable on an interval  $I$ , and if  $|f'(x)| \leq M$  for all values of  $x$  in  $I$ , then (10)  

$$|f(x) - f(y)| \leq M|x - y|$$
for all values of  $x$  and  $y$  in  $I$ . Use this result to show further that  

$$|\sin x - \sin y| \leq |x - y|.$$
- (b)** Prove that if  $x = x(t)$  and  $y = y(t)$  are differentiable at  $t$ , and if (10) **(20)**  
 $z = f(x, y)$  is differentiable at the point  $(x, y) = (x(t), y(t))$ , then  
 $z = f(x(t), y(t))$  is differentiable at  $t$  and  

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$
where the ordinary derivatives are evaluated at  $t$  and the partial derivatives are evaluated at  $(x, y)$ .
- Q. 5. (a)** Evaluate the double integral (10)  

$$\iint_R (3x - 2y) dx dy$$
- (b)** Where  $R$  is a region enclosed by the circle  $x^2 + y^2 = 1$ . (10) **(20)**  
Find the area of the region enclosed by the curves  

$$y = \sin x, \quad y = \cos x, \quad x = 0, \quad x = 2\pi.$$

## PURE MATHEMATICS

- Q. 6. (a)** Find an equation of the ellipse traced by a point that moves so that the sum of its distance to (4,1) and (4,5) is 12. (10)
- (b)** Show that if  $a, b$  and  $c$  are nonzero, then the plane whose intercepts with the coordinate axes are  $x = a, y = b,$  and  $z = c$  is given by the equation. (10) (20)

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

### SECTION-C

- Q. 7. (a)** Prove that a necessary and sufficient condition that  
 $w = f(z) = u(x, y) + iv(x, y)$   
be analytic in a region  $R$  is that the Cauchy-Riemann equations  
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$
are satisfied in  $R$  where it is supposed that these partial derivatives are continuous in  $R$ . (10)
- (b)** Show that the function  $f(z) = \bar{z}$  is not analytic anywhere in the complex plane  $Z$ . (10) (20)
- Q. 8. (a)** Let  $f(z)$  be analytic inside and on the boundary  $C$  of a simply-connected region  $R$ . Prove that (10)

$$f'(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2} dz.$$

- (b)** Show that

$$\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)^2} = \frac{5\pi}{32}. \quad (10) \quad (20)$$

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Reach out to us @ [info@thinked.co](mailto:info@thinked.co)  
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[writeforthinked@thinked.co](mailto:writeforthinked@thinked.co)