

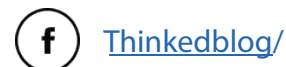


CSS Past Paper

# Purer Mathematics

(2020)

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FEDERAL PUBLIC SERVICE COMMISSION  
COMPETITIVE EXAMINATION-2020  
FOR RECRUITMENT TO POSTS IN BS-17  
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100
<b>NOTE: (i)</b> Attempt FIVE questions in all by selecting TWO Questions each from SECTION-A&B and ONE Question from SECTION-C. ALL questions carry EQUAL marks. <b>(ii)</b> All the parts (if any) of each Question must be attempted at one place instead of at different places. <b>(iii)</b> Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper. <b>(iv)</b> No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed. <b>(v)</b> Extra attempt of any question or any part of the attempted question will not be considered. <b>(vi) Use of Calculator is allowed.</b>	

SECTION-A

- Q. 1. (a)** Let  $G$  and  $G'$  be two groups and  $f : G \rightarrow G'$  be a homomorphism then prove the following: (10)
- (i)  $f(e) = e'$  where  $e$  and  $e'$  are the identities of  $G$  and  $G'$  respectively
- (ii)  $f(a^{-1}) = [f(a)]^{-1}, \forall a \in G$
- (b)** Prove that every homomorphic image of a group is isomorphic to some quotient group. (10) (20)
- Q. 2. (a)** A ring  $R$  is without zero divisor if and only if the cancellation law hold. (10)
- (b)** Prove that arbitrary intersection of subrings is a subring. (10) (20)
- Q. 3. (a)** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear transformation defined by (10)
- $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3, x_2 + x_3)$ . Find a basis and dimension of Range of  $T$ .
- (b)** Prove that every finitely generated vector space has a basis. (10) (20)

SECTION-B

- Q. 4. (a)** Find the critical points of  $f(x) = x^3 - 12x - 5$  and identify the open intervals on which  $f$  is increasing and on which  $f$  is decreasing. (10)
- (b)** Find the horizontal and vertical asymptotes of the graph of  $f(x) = -\frac{8}{x^2 - 4}$  (10) (20)
- Q. 5. (a)** Calculate  $\int \frac{-2x + 4}{(x^2 + 1)(x - 1)^2} dx$ . (10)
- (b)** Find  $\frac{\partial w}{\partial x}$  at the point  $(x, y, z) = (2, -1, 1)$  if  $w = x^2 + y^2 + z^2, z^3 - xy + yz + y^3 = 1$  (10) (20) and  $x$  and  $y$  are the independent variables.
- Q. 6. (a)** Determine the focus, vertex and directrix of the parabola  $x^2 + 6x - 8y + 17 = 0$  (10)
- (b)** Find polar coordinates of the point  $p$  whose rectangular coordinates are (10) (20)
- $(3\sqrt{2}, -3\sqrt{2})$

PURE MATHEMATICS

SECTION-C

**Q. 7. (a)** Show that  $(\cos \theta + i \sin \theta)^n = \cos(n \theta) + i \sin(n \theta)$  for all integers  $n$ . (10)

**(b)** Find the  $n$ ,  $n$ th roots of unity. (10) **(20)**

**Q. 8. (a)** Find the Taylor series generated by  $f(x) = \frac{1}{x}$  at  $a = 2$ . Where, if anywhere, (10)

does the series converge to  $\frac{1}{x}$ ?

**(b)** Show that the p-series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$ , ( $p$  a real constant) converges if  $p > 1$ , and (10) **(20)**  
diverges if  $P < 1$

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