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Cambridge O Level

Additional Mathematics

4037

(May/June 2018)

ADDITIONAL MATHEMATICS

Paper 4037/11
Paper 11

Key messages

If it is stated in a question that a calculator must not be used, candidates should be aware that they must show all steps of their working. Questions starting with the word 'Hence' should use a method making use of the result from the previous part.

General comments

The paper provided a good range of responses showing that many candidates understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues. Except for a very few candidates who attached large numbers of additional pages, candidates made good use of the space provided on the paper and responses were generally straightforward to mark with a good standard of presentation.

Comments on specific questions

Question 1

Nearly all candidates knew how to approach this question and the majority obtained a quadratic equation in either x or y by substituting either $y = x + 4$ or $x = y - 4$. Solutions of the quadratic equation were well done but many candidates found the value of just one variable and did not go on to find the other.

Answer: $x = 4$, $y = 8$ and $x = -2$, $y = 2$

Question 2

Nearly all candidates found the gradient of the line joining (1, 3) and (4, –5) and many of those correctly found the value of the perpendicular gradient. Not all candidates realised that the mid-point was required to form the equation of the perpendicular bisector and lost marks when they used the given points in their equation. There were many good solutions that omitted to show their equation in the form asked for in the question.

Answer: $6x - 16y - 31 = 0$

Question 3

Successful candidates showed work on the graphs, such as diagonal and vertical lines, and clearly considered how the given properties related to graphical representations. Candidates who guessed were usually unsuccessful as extra ticks were penalised.

Answer:

A	B	C	D
	✓		
		✓	✓
		✓	
✓			

Question 4

(i) Candidates were expected to approach this question by equating *b* to the amplitude and calculating *c* from the value given for the period. They could then find *a* using the given point. Those who started by equating *a* to the amplitude could not earn marks. Another common error was to assume that *c* was $\frac{\pi}{3}$. Candidates would benefit from practice with this type of question.

(ii) Although the question required a sketch graph, sufficient structure was provided for candidates to submit a carefully drawn graph. Candidates who had an incorrect equation from part **(i)** often produced a graph with three complete cycles. A common mistake in otherwise well drawn graphs was to have the final point at $(\pi, 0)$ rather than $(\pi, -2)$.

Answers: **(i)** $a = -2$, $b = 4$, $c = 6$

Question 5

(i) Although many candidates understood what was required here, some just said it was a constant and others thought it was the rate of increase.

(ii) There were many good solutions with most candidates finding e^{2k} and then using natural logs to find *k*. There were also good solutions that started with taking natural logs of both sides of the original expression, but this method was generally less successful. Candidates did not always evaluate their final answer and there were some who rounded prematurely, thus affecting their answer to the next part.

(iii) Nearly all candidates knew how to use their value of *k* to find a final answer.

Answers: **(i)** The number of bacteria at the start of the experiment **(ii)** 1.61 **(iii)** 100 000

Question 6

(a) Candidates were expected to use the change of base formula and simplify the first term to $\log_3 p$ and then use the laws of logarithms. Candidates who arrived at a correct result using incorrect intermediate steps could gain no credit as they had made compensating errors. Candidates who used $\log_3 2^{\log_2 p}$ were usually unable to simplify to $\log_3 p$ to arrive at the final expression.

(b) Some candidates did not recognise the equation as a quadratic one. Most successful candidates substituted a letter for $\log_a 5$ and solved the resulting quadratic equation. Most then found $\log_a 5 = 1$ straightforward to solve but there was less success solving $\log_a 5 = 3$. Some candidates rounded their answer prematurely which was possibly the result of a trial and improvement approach in this final step.

Answers: **(a)** $\log_3 pq$ **(b)** 5, 1.71

Question 7

(i) This question was well answered by most candidates.

(ii) Candidates should be aware that their answer in part **(i)** had to be used in their method of solution in this part. Candidates who used algebraic methods were not awarded marks as a matrix method was required. Most candidates who pre-multiplied by an inverse matrix did so accurately but a large proportion of candidates did not relate the given equations to the matrix in the previous part. Candidates who used matrices usually set out their solutions clearly and well.

Answers: **(i)** $\frac{1}{2}\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$ **(ii)** x = 7.25, y = 13.25

Question 8

(a) Most candidates obtained $2\mathbf{i} - 3\mathbf{j}$ from expanding and collecting like terms. Some errors occurred in dealing with the minus sign in the second bracket when expanding. Many candidates left their answer as $2\mathbf{i} - 3\mathbf{j}$. Of those who went on to square and add, a significant number went no further and did not divide by $\sqrt{13}$ to obtain a unit vector.

(b) Candidates had to appreciate that the resultant velocity was in the direction of *AB* and therefore *v* was the hypotenuse of a right-angled triangle. This model of the situation was required for both parts and candidates who misunderstood the question could not earn marks in either part.

(i) Most candidates tried to use Pythagoras' theorem but not all had identified *v* as the hypotenuse. Those using a correct diagram usually went on to find the correct answer.

(ii) Most candidates used trigonometry, but its use in this part had to be based on a correct diagram. Candidates using a correct trigonometric ratio were usually successful in obtaining a correct answer.

Answers: **(a)** $\frac{2\mathbf{i} - 3\mathbf{j}}{\sqrt{13}}$ **(b)(i)** 3.00 **(ii)** 24.6°

Question 9

Candidates found both parts of this question difficult and would benefit from practice in this type of question.

(i) Although correct binomial coefficients were seen, the application of the theorem for an expansion of an expression with two algebraic terms proved difficult. Dealing with the fraction was an added complication which was found difficult by many candidates. Candidates should be aware that the final answers had to be fully simplified and not left as fractions. Some good responses were marred by carelessness with the signs and mistakes in arithmetic.

(ii) Many candidates correctly expanded $\left(\frac{1}{x^2} + 1\right)^2$ and some good solutions were seen. Most candidates tried to form at least one product but not all had suitable terms from part **(i)** to form three appropriate products leading to terms in x^4.

Answers: **(i)** $256x^8 - 64x^6 + 7x^4$ **(ii)** 135

Question 10

Candidates were instructed not to use a calculator in this question. Care had to be taken to show all working leading to their answers. Solutions showing no working or minimal working were unable to be awarded all of the marks.

(a) Most candidates knew how to rationalise and most answered this question well. Those with missing and incorrect working could not access all of the marks. It was obvious that a few had used a calculator to obtain a correct final answer.

(b) Many correct answers which had been correctly obtained were seen. However, again, candidates giving a correct answer with incorrect, incomplete or no working were not awarded full marks. Some candidates had difficulty dealing with $\left(\sqrt{2}\right)^7$ and misunderstood the laws of indices.

(c) Nearly all candidates successfully rearranged the given equation as a quadratic equation. The most common approach to this question was to use the formula. Substitution in the formula was nearly always correct but expressing $\sqrt{(2+16)}$ in terms of $\sqrt{2}$ was rarely dealt with adequately. Completing the square and factorisation were used by some candidates, but they also required the candidate to show sufficient working. Solutions in terms of $\sqrt{2}$ can easily be obtained from a calculator and examiners had to be convinced that candidates were not working back from those.

Answers: **(a)** $\sqrt{5}$ **(b)** $8\sqrt{6}$ **(c)** $\sqrt{2}$, $-2\sqrt{2}$

Question 11

Some candidates confused techniques of differentiation with those for integration and therefore scored poorly in both parts.

(i) This question was well answered. Most candidates differentiated correctly and went on to find a correct value of x and to earn full marks.

(ii) There were two approaches to this question, but the most common was to find the area under the curve and the area of the trapezium separately and subtract. Candidates using this method often had either misunderstood the question or forgotten the need to find the area of the trapezium and found the area under the curve only. Those forming an equation for the line and subtracting the equation of the curve before integration were generally successful but there was a greater chance of sign errors with this method. In either method, good attempts at integration were made and application of limits was well executed.

Answers: **(i)** $x = \frac{3}{2}$, $y = 36$ **(ii)** 12

Question 12

A good number of correct solutions were seen with candidates coping well with a long, less structured question. Most candidates knew that integration was necessary and most made good attempts at the required integrations. To obtain the equation of the curve use had to be made of the constants of integration. Candidates who had correctly used $\frac{dy}{dx} = 6$ when $x = \frac{9}{2}$ to find a first constant did not always integrate the full expression obtained for $\frac{dy}{dx}$ to find y. Many candidates did not score beyond their attempts at integration as they either had no constants of integration or made incorrect substitutions involving $\frac{9}{2}$, 6 and $\frac{2}{3}$. A significant number of candidates misunderstood the question and used the gradient of 6 to create a straight line equation.

Answer: $y = \frac{1}{3}(2x-5)^{\frac{3}{2}} + 4x - 20$

ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

Candidates are reminded that their answers should be given correct to 3 significant figures unless stated otherwise. Too many candidates 'lost' marks by giving 2 significant figure answers of truncated answers. It is essential that solutions are set out clearly. On occasion, a candidate may run out of working space for a particular question. In this case extra paper should be requested in order to complete the solution clearly.

General comments

The paper provided a good range of responses showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. There appeared to be no timing issues.

Comments on specific questions

Question 1

(i) A response in either radians or degrees was acceptable. Many candidates gave a response of $120°$ or $\frac{2\pi}{3}$, clearly thinking that functions of sine and cosine have the same period as tangent.

(ii) Many candidates did not produce clear sketches. They should be aware that a graph of $y = 1 + \tan 3x$ has asymptotes and that these asymptotes should really be sketched in using vertical dotted lines at the appropriate points. It was expected that a discontinuous curve be drawn, approaching these asymptotes. The curve was expected to start at $(0°, 1)$ and end at $(180°, 1)$. Completely correct and acceptable sketches were rare.

Answers: **(i)** $\frac{\pi}{3}$ or $60°$

Question 2

Sign errors when dealing with the simplification of bracketed terms cost many candidates full marks. Most candidates used a correct approach of equating the equation of the line and the equation of the curve. Problems arose when simplifying this result to the form of a 3-term quadratic equation, equated to zero. Most errors were made with the coefficient of the *x*-term and this then caused problems when making use of the discriminant in order to find the critical values. Provided the correct critical values had been found using a correct method, most candidates then went on to find a correct set of values for *x*.

Answer: $-13 < k < 11$

Question 3

There were a great many correct responses for this question. Most candidates realised that the equation needed to work with was of the form $e^y = mx^2 + c$. Recognition that the gradient was equal to 1 followed by a correct substitution using one of the given points usually resulted in the equation $e^y = x^2 - 2$. Many candidates, having reached this point often omitted to find *y* in terms of *x* correctly, with answers of the form $y = \ln x^2 - 2$ and $y = \ln x^2 - \ln 2$ being all too common.

Answer: $y = \ln\left(x^2 - 2\right)$

Question 4

Provided candidates were familiar with the syllabus requirements for Kinematics, many were able to score highly.

(i) A solution of $e^3 = 5t + 3$ was expected, correct to 3 significant figures. An exact answer had not been asked for and as the question related to a real-life situation, a non-exact answer was expected. Many candidates lost marks by giving an answer correct to 2 significant figures.

(ii) There was a good response to this part of the question, with most candidates recognising that differentiation was needed and producing this correctly.

(iii) Many candidates did not appreciate the situation in a real-life context and many completely incorrect responses were given. All that was expected was for the candidate to state that time can never be negative so the denominator of the equation $v = \frac{5}{5t+3}$ is always positive and so the velocity is always positive. This response did depend on the candidates having a correct form for their answer to part **(ii)**. Other correct arguments were acceptable.

(iv) Having obtained a correct response to part **(ii)**, many candidates than attempted to differentiate their result for the velocity as a quotient, with respect to time. Unfortunately many did not recognise that the derivative of 5, with respect to time, is zero, so incorrect responses of $-\frac{22}{9}$ were common. It was intended that candidates make use of the chain rule for differentiation.

Answers: **(i)** 3.42 **(ii)** $\frac{5}{3}$ **(iv)** $-\frac{25}{9}$

Question 5

(i) Many correct responses were seen. Candidates were asked to find the value of each of the coefficients *a*, *b* and *c*. Many did this correctly but a number of candidates chose to multiply their coefficients by 3 in order to have integer responses. A number of candidates, who, having a correct expansion, stated that $c = 10$.

(ii) Correct expansions of $\left(2 + 9x\right)^2$ gained most candidates a mark. Many recognised that they need to consider three terms that can be obtained from making use of their expansion from part **(i)** together with that of $\left(2 + 9x\right)^2$. Even if an error had been made in part **(i)**, 2 of the 3 marks available could be gained. It was noted that most candidates are now aware of the meaning of the phrase ‘independent of *x*’.

Answers: **(i)** $a = 243$ $b = –45$ $c = \frac{10}{3}$ **(ii)** – 378

Question 6

Many correct responses evidenced that many candidates recognised the need to make use of both the chain rule and differentiation of a quotient. Some correct algebraic manipulation of the result equated to zero was needed. It was often at this point that errors occurred.

Answer: $\left(3, \sqrt{5}\right)$

Question 7

(i) There were very few incorrect responses seen.

(ii) Many candidates did not appreciate what exactly was meant by 'double in size'. It was intended that the answer to part **(i)** was used to obtain the equation $e^{\frac{t}{4}} = 2$. Many candidates, having got this far, were often unable to solve this equation for *t*. Of those candidates that did use a correct method of solution, answers correct to 2 significant figures meant that the last accuracy mark could not be gained.

(iii) There were very few incorrect responses seen.

Answers: **(i)** 1000 **(ii)** 2.77 **(iii)** 7390

Question 8

(a) Many candidates were able to obtain full marks for this part of the question, having used the correct trigonometric identity together with the correct solution of the resulting quadratic equation. Those candidates who incorrectly included $\theta = 180^\circ$ as a solution for $\sin\theta = 1$ could not gain the final mark. Candidates are to be reminded that angles are to be given correct to 1 decimal place and that answers of 161° were thus not acceptable.

(b) Many candidates did not consider solutions to a correct equation of $\tan 2\phi = \sqrt{3}.$ They were usually able to obtain 2 marks for the correct positive angle. A correct order of operations was used by most. Consideration of negative angles needs to be practiced by centres as it is a syllabus requirement.

Answers: **(a)** $19.5^\circ,\ 160.5^\circ,\ 90^\circ$ **(b)** $-\frac{\pi}{3}, \frac{\pi}{6}$

Question 9

(a) (i) Very few incorrect responses were seen.

(ii) It was essential that candidates made use of the response to part **(i)** by writing 3 as $\lg 1000$ and then writing $2\lg b$ as $\lg b^2$ before making any attempt at further simplification. Many candidates did not recognise the importance of part **(i)** but were able to gain method marks for making correct use of the laws of logarithms.

(b) (i) Most candidates formed a correct quadratic equation and solved it correctly.

(ii) Many candidates did not appreciate the use of the word 'Hence' and attempted to start the question again, not noticing the similarity between the equation in part **(i)** and the one they were working with in part **(ii)**. It was intended that candidates recognise the connection and write $x = \log_4 a$ and then make use of the solutions obtained in part **(i)**.

Answers: **(a)(i)** 1000 **(ii)** $\lg\frac{1000a}{b^2}$ **(b)(i)** 2, 3 **(ii)** 16, 64

Question 10

When instructed not to use a calculator, it is essential that candidates show each step of their working clearly and concisely making no omission. Missing steps in solutions of this type imply that a calculator may have been used and subsequent marks will not be available.

(i) Most candidates recognised the need to use the cosine rule and were able to write down a correct equation. It was expected that they show expansions of the relevant terms before collecting like terms together. Some candidates did not provide enough detail and were thus not able to gain full marks for this part.

(ii) Far fewer correct solutions were seen to this part of the question. It was clearly evident from the work of many of the candidates that calculators had been used to simplify intermediate steps. Most candidates were able to write down a correct sine rule, making use of their answer to part **(i)**. It was then necessary to use this equation to obtain an expression for $\operatorname{cosec} ACB$ and then attempt rationalisation. As in part **(i)**, lack of essential working meant that full marks could not be awarded, especially as the form of the answer had been given.

Answers: **(i)** $\sqrt{123}$ **(ii)** $\dfrac{2\sqrt{41}\left(4\sqrt{3}+5\right)}{23}$

Question 11

This was a completely unstructured question which needed candidates to make use of their problem solving skills and formulate a method of solution using the appropriate steps in the appropriate order. There were many correct solutions and many solutions that did not gain the final accuracy mark as the final answer was not given in the required form. Method marks were available for the candidates who made errors in some of the initial stages but used a correct approach in the final stages.

It was required that candidates find the coordinates of the points *A* and *B* first. Some errors in the differentiation of the exponential function resulted in incorrect coordinates for the point B. Integration of the given function was also necessary and some candidates did not integrate the given function as $\dfrac{e^{4x}}{8}+\dfrac{3}{8}$. It was essential to show all working as requested, including the substitution of limits after integration. It was evident that some candidates had made use of the definite integral function on their calculator. These candidates could not gain all the marks as they did not show sufficient essential working as requested.

Answer: $\dfrac{e}{32}$

Question 12

(a) It was essential that candidates attempt to write each term in terms of factors of 2 and/or 3 where appropriate. This was not often done, resulting in fewer correct solutions than expected. The powers of 2 yielded the value of *p* which could then be used in the powers of 3 to yield the value of *q*.

(b) Most candidates made use of the given substitution to obtain a quadratic equation in *u* which gave the solutions $u=-1,\ u=-3.$ Many candidates stopped at this point being unable to solve these equations in order to find the possible values of x. Some candidates thought that they had to take the cube root of –1 and –3, rather than simply cube –1 and –3 to obtain the required values of *x*.

Answers: **(a)** $p=\dfrac{1}{4}\ q=\dfrac{3}{4}$ **(b)** –1, – 27

ADDITIONAL MATHEMATICS

Paper 4037/21
Paper 21

Key messages

In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded. In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. In such questions candidates are encouraged to use consistent notation such as using the same variable throughout a solution and should avoid replacing a function of a variable with the variable itself. In questions that state that a calculator should not be used, omitting method steps often results in full credit not being given for a solution. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit. Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator, without showing any working. When a diagram or graph is required then they should be completed in full and as accurately as possible taking note of specific features which are requested in the question.

General comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given. More credit was likely to be given when a clear sequence of steps was evident. Quoting a formula which referred to only part of the previous line then applying it on the next line led to candidates confusing themselves.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates need to improve their reading of questions and keep their working relevant in order to improve. Candidates should also read the question carefully to ensure that, when a question requests the answer in a particular form, they give the answer in that form particularly when the question states that an exact answer is required. Candidates should ensure also that each part of a question is answered and the answer clearly identified. When a candidate uses the blank page or an additional booklet they should make it clear which question their work relates to. It is not possible in most cases to connect work otherwise to a specific question which can lead to the loss of potential credit. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. They should also be aware of the need to use the appropriate form of angle measure within a question. When a question indicates that a calculator should not be used, candidates must realise that clear and complete method steps should be shown and that the sight of values clearly found from a calculator will result in the loss of marks.

Where an answer was given and a proof was required, candidates needed to fully explain their reasoning. Omitting method steps in such questions resulted in a loss of marks. Working from both sides and so treating an identity like an equation is not a valid way to prove a given result. Candidates should work from the left hand side to arrive at the result stated on the right hand side or begin with a quotable formula and rearrange it correctly.

Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

Candidates should be advised that any work they wish to delete should be crossed through with a single line so that it can still be read. There are occasions when such work may be marked and it can only be marked when it is readable.

Comments on specific questions

Question 1

The answers earning most credit were those which showed the candidate's clear understanding of the symbols used in set notation. Where a description was required, those who used set terminology were more likely to succeed than those who attempted to give their answers using common English phrases. Where set notation was required, there was much evidence of using the wrong symbol or trying to interpret the statement given bit by bit.

(i) (a) This was very well answered, with few candidates making errors. The most common error seen was to state *A* and *B* the wrong way round. When candidates tried to use terminology other than *subset* answers were frequently unclear or imprecise.

(b) It was rare to see the statement *A and C are mutually exclusive* given as the answer. It was more common to see *A intersection C is the empty set* although some did lose credit by using *zero* rather than *the empty set*. There were attempts where set *B* was referred to rather than *C*. As with the previous part, descriptive explanations were often unclear or ambiguous, implying that *A* and *C* were also empty.

(ii) (a) This part met with mixed success. There were many correct statements but intersection rather than union was fairly commonly used. Another common answer was to state that *A* and *B* also had three members as separate parts of the final answer.

(b) This part proved more challenging with the answer involving the intersection of sets *A* and *C′* rarely expressed correctly. Candidates usually gave equivalent answers involving *x* being a member of *A*, *x* being a member of *C′* or *x* not being a member of *C*. Frequently, notational problems such as using subset rather than element or omitting brackets caused a loss of possible credit.

Answers: **(i)(a)** *A* is not a proper subset of *B* **(b)** *A* and *C* are mutually exclusive **(ii)(a)** $\mathrm{n}(A \cup B) = 3$
(b) $x \in (A \cap C')$

Question 2

(i) This was one of the best attempted parts of the entire paper with most candidates differentiating the ln function correctly. A few candidates gave the numerator incorrectly as 1 and a few were unable to apply the appropriate format.

(ii) By contrast, this part was poorly attempted with many candidates unable to build on their success in part **(i)**. The concept of using the connection between small increments and differentiation was being tested here. Although many recognised this and noted the hence in the question, it was usual to apply the *y* value to the derivative and the corresponding *x* value was rarely found. It was fairly common to calculate two separate *x* values and find their difference. This frequently involved replacing *x* in their derivative with two *y* values.

Answers: **(i)** $3 \times \frac{1}{3x-1}$ **(ii)** 0.05

Question 3

This was a well-answered question with many candidates realising that permutations were required and achieving full marks. The most common error was to use combinations. Candidates who did this had not understood that, in a password, the order of the characters matters. Part **(iii)** was often not fully correct as, while candidates usually knew how to form the permutations required as a block of upper-case letters and a block of lower-case letters, the realisation that the blocks could appear in two orders was omitted. Another error here was to multiply 4! by 3! forgetting that there were still four lower-case letters to choose from.

Answers: **(i)** 3 991 680 **(ii)** 1 330 560 **(iii)** 1152

Question 4

(i) This question was well answered by most candidates. The most common approach was to use the factor theorem and the remainder theorem, although long division was also often seen. When b was incorrect, candidates had usually made an arithmetic slip. Few errors in finding a were made when candidates used the factor theorem. Other methods were more likely to contain errors. This was especially the case when candidates used long division and had to do algebraic work at the end to verify the value.

(ii) Those who carried out the appropriate steps in full rarely made an error in their algebra. The requirements of the question clearly stated that factorisation needed to be carried out in order to gain full credit. There were a few candidates who wrote down the cubic followed immediately by its three linear factors with no working. In a question like this, candidates must realise the need to show all working. Finding the roots and working back to the factors did not meet the requirements of the question. Other candidates found a correct quadratic factor and then factorised it correctly without ever showing all three linear factors together. As the question required $p(x)$ to be factorised, this resulted in a loss of marks.

Answers: **(i)** $b = -30$ **(ii)** $(2x+1)(x-3)(x+4)$, $x = -0.5, 3, -4$

Question 5

(i) The majority of candidates showed a good understanding of how to find an inverse function showing clear steps leading to a correct answer. Correct methods with errors usually arose due to arithmetic slips or algebraic slips. Only a very small minority gave a partial method, with the final answer in terms of y rather than x. Misunderstanding the notation leading to differentiation was seen but very rarely.

(ii) There were few correct answers to this part and many omissions. Candidates often tried to manipulate their answer to part **(i)** rather than considering the range of $f(x)$. When candidates did identify that this must be positive, their given answer often was in terms of y or included zero.

(iii) This part, which required a significant amount of algebraic manipulation was attempted very well on the whole. The concept of finding the composite function $ff(x)$ was understood by the majority with relatively few candidates incorrectly squaring the given function. Care needed to be taken by those candidates who worked with the denominator separately as sometimes the complete fraction was omitted at the end. Most slips occurred due to sign errors, usually when multiplying out a bracket, although a few candidates incorrectly changed a sign having previously arrived at a correct solution.

Answers: **(i)** $f^{-1}(x) = \frac{1}{2}\left(\frac{1}{x}+5\right)$ **(ii)** $x > 0$ **(iii)** $\frac{2x-5}{-10x+27}$

Question 6

This question was completed very well by the majority of candidates, with very few slips in any part.

Even the weakest candidates made good progress with this question demonstrating a sound knowledge of appropriate formulae for arc length and sector area. A few candidates introduced a spurious factor of π into their formulae. A few other candidates chose to work in degrees rather than radians; they lost marks because of a lack of accuracy in the final answer. Part **(iii)** was the part of the question where candidates were most likely to lose credit. This was commonly because they took the area of sector *DOC* to be 140 rather than 180 or erroneously subtracted a correct answer from 16 as an additional step.

Answers: **(i)** 2.5 **(ii)** 320 **(iii)** 12

Question 7

(i) The use of the product rule was well-understood and correctly applied by most candidates. Occasionally candidates made errors when finding the derivative of tan x. Some candidates made this more complicated than necessary by converting the tan x to $\frac{\sin x}{\cos x}$, which led to product rule within quotient rule, or even product rule within product rule. These approaches were often more likely to contain errors. There were some who gave a single term as the answer, clearly not recognising the product.

(ii) This was done well by a large number of candidates with the majority recognising this as the quotient rule. Some candidates attempting to use the quotient rule reversed the terms in the numerator or did not square the denominator. Some candidates were unable to differentiate the exponential term correctly or misinterpreted it as $e^{3x}+1$. The product rule was seen on a number of occasions.

In both parts, simplification of the answer was not demanded in the question and further errors after a correct application of a rule were therefore ignored.

Answers: **(i)** $4\tan x + 4x\sec^2 x$ **(ii)** $\frac{(x^2-1)(3e^{3x+1})-2xe^{3x+1}}{(x^2-1)^2}$

Question 8

(i) The conversion to linear form was generally accurate. The most common error was to incorrectly bring the power of n to the front of the log before separating the expression such as $\ln(ax^n)$ into the sum of two log terms. Most candidates took the natural logarithms approach, with a small proportion taking logs to the base 10 or x.

(ii) Those candidates who could link n and a appropriately to the gradient and intercept of the given line had a basis for success. Even when this connection was properly understood, marks could often not be awarded since the points candidates used to find the gradient and/or intercept were data points, not points chosen from the line. Candidates need to understand that in this type of question, the line encapsulates the relationship between the variables and takes account of all the experimental data that has been gathered. Working with particular data points ignores this relationship and therefore is an invalid approach.

(iii) Partial credit was available here for correct use of the candidate's values from part **(ii)**, although insufficient clarity in the method often led to this not being awarded. Few candidates used the most direct method of reading the value of ln x from the graph corresponding to ln 50.

Answers: **(i)** $\ln y = \ln a + n\ln x$ or $\lg y = \lg a + n\lg x$ **(ii)** $n = -0.2$ to -0.3, $a = e^{4.7}$ or 110 **(iii)** 22

Question 9

(i) The method of completing the square was generally well understood although not always carried out accurately. The most common error made was in finding *r*. Sign slips and incorrect formats were not uncommon. When a form for an answer is given in the question candidates should ensure that their final answer fits this.

(ii) Sketches varied enormously in quality. The best showed accurately the shape and position of the component parts. The weakest seemed for the most part to show appreciation of the fact that a modulus graph can have no part below the *x*-axis. The position of the maximum also posed a difficulty with some candidates placing it on the *y*-axis. Even the most precisely drawn sketches did not always earn full credit because the instruction given in the question about axis crossing points being shown was ignored, particularly the *y*-intercept. Candidates should be aware of the need for the curvature of the graph to be appropriate at all parts, especially at the cusps or extremities of the graph, as this is part of the skill that is being assessed.

(iii) This was generally one of the most poorly attempted parts of the paper. Very few candidates made the link with part **(i)** and the maximum point in part **(ii)**. As a result, much irrelevant algebraic work was often carried out. When the connection was made, the given inequality was seldom fully correct with the lower limit frequently omitted and/or the upper limit linked to an incorrect inequality.

Answers: **(i)** $5\left(x-\frac{7}{5}\right)^2-\frac{64}{5}$ **(iii)** $0<k<\frac{64}{5}$

Question 10

(i) Many candidates were able to differentiate correctly to find *v*, but few arrived at the correct value for *t*, many giving the answer in degrees, or abandoning the solution after finding $t = 0$. A minority got the correct exact answer as required but then went on to express it as a rounded decimal. Some candidates incorrectly equated *s* to zero. Integration instead of differentiation was rarely seen.

(ii) Almost all candidates earned only one of the three marks available through not appreciating that the particle changed its direction of travel between $t=\frac{\pi}{4}$ and $\frac{\pi}{2}$. Understandably this was missed if $t=\frac{\pi}{3}$ had not been found in part **(i)** but, even when this had been found, its significance for part **(ii)** was almost always overlooked. A simple sketch of the motion would have helped. It was not uncommon to see the answer for part **(i)** integrated, as some candidates did not see that they could use the given expression for *s*. Some candidates integrated the given expression for *s*.

(iii) Differentiation to find acceleration from velocity was well done on the whole with some candidates who had not found the velocity earlier showing both stages here. Many candidates differentiated again and equated rate of change of acceleration to zero to find the maximum. Many were able to spot that the minimum value of the cosine function was needed to give the maximum acceleration. There were instances where a lot of additional algebra took place resulting in an incorrect answer. A very common mistake was to give –9 as the maximum.

Answers: **(i)** $\frac{\pi}{3}$ **(ii)** 1.29 **(iii)** 9

Question 11

(i) This question required the correct use of a trigonometric identity followed by the solution of a quadratic in sin x and the solution of the resulting equation to give solutions in degrees. The majority of candidates were able to make some progress, with most working through the full process. There were some sign errors in the first step but most who reached the correct quadratic solved it correctly, usually by factorisation. Candidates are advised to use a letter other than x to replace sin x when making the expression easier to factorise. Using the same letter could cause confusion and in some cases led to the final step being omitted. When an equation has two strands it is advisable that candidates list their answers at the end of their solution as it is sometimes difficult to separate working values from answer values. Care should also be taken to give answers to the correct accuracy as indicated in the rubric on the front cover.

(ii) There were very few fully correct answers to this question. Almost always this was because a solution was lost after dividing through by sin 2y which appeared on both sides of the equation after writing tan 2y in terms of sin 2y and cos 2y. When solving the remaining equation in cos 2y it was not uncommon for only one solution to be given. As a result of premature approximation in the working, one of the final answers was often rounded incorrectly or given to just 2 significant figures. Candidates are advised to work in radians, as working in degrees is more likely to result in an inaccurate final answer in radians when converted.

Answers: **(a)** 30°, 150°, 191.5°, 348.5° **(b)** $\frac{\pi}{2}$, 0.361, 2.78

Question 12

(i) The proof was generally the part which was answered best, usually by candidates who employed simple trigonometry in one of the right-angled triangles. There were some more complex attempts which were, as a result, frequently unsuccessful. Poor notation was an issue for some candidates who used x for half the base then doubled their answer without justification. As always when the answer is given the solution needs to be completely correct. The volume was quite often twice what it should have been as a result of an error in finding the cross-sectional area of the water at the end of the container.

(ii) (a) Candidates who set their answers out clearly and in an appropriate order were most likely to succeed. There were many correct statements of the chain rule linking the volume, the height and time, although it was difficult to follow what variables were being used for which quantities in some cases. Some candidates confused $\frac{\mathrm{d}h}{\mathrm{d}V}$ with $\frac{\mathrm{d}V}{\mathrm{d}h}$ and many other candidates rearranged their answer to part **(i)** to get h in terms of V in order to differentiate which often led to errors in simplification at some stage.

(b) This required a further use of connected rates of change and entailed multiplying their answer to part **(ii)(a)** by $2\sqrt{3}$. Despite their problems in part **(ii)(a)**, there were many candidates who could attempt this part correctly. However a fully correct exact answer was rarely seen. Whilst candidates had fully correct working for both of these parts, the final answer was often inaccurate. Working with exact values throughout was a good strategy as it would have resulted in an exact final answer.

Answers: **(i)** $V = 5\sqrt{3}h^2$ **(ii)** 0.115 **(iii)** 0.4

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

To be successful in this examination, candidates need to give clear, logical answers to questions and show enough clear method so that marks can be awarded. Candidates who omit to show key method steps in their solution, perhaps because they have used a calculator or because they have combined steps, risk losing a significant number of marks should they make an error. This is the case whether or not the use of a calculator is permitted for a particular question. Candidates should ensure that their answers are given to at least the accuracy required in a question. When no particular accuracy is requested, candidates should make sure that they follow the instructions printed on the front page of the examination paper. Candidates need to know that the accuracy required for angles in degrees varies from those in radians and this information is also available on the front page of the examination paper. Candidates should take care with the accuracy of working values and understand that, in order to give an answer correct to 3 significant figures, their working values need to be more accurate.

General comments

Many candidates seemed to be very well prepared for this examination. They showed good understanding of basic concepts and were able to apply techniques successfully.

Candidates whose presentation was good, indicating a logical thought progression, often scored more highly. Other candidates need to understand that, when their work is difficult to follow it is difficult for credit to be given. Sometimes candidates miscopy their own writing when their presentation is poor and accuracy is, unnecessarily, lost. Some candidates used extra sheets of paper to carry out 'rough calculations'. These calculations were usually then deleted and annotated as rough work. This work was often poorly presented and difficult to assign to particular questions. On occasion, it is possible to award marks for rough work that has been deleted and not replaced. This can only be done if the work is readable. Candidates who use rough paper can also miscopy their own work. For these reasons, candidates should write all of their working as part of their main solution to a question and use extra paper only to rewrite or correct a solution to a question. Extra paper for rough working should not be used.

Many candidates showed all their method clearly, step by step. Other candidates could improve by showing a complete method, rather than a partial method. This is essential if a question asks candidates to 'Show that…'. This instruction indicates that information about the answer has been given. In these cases, the marks are awarded for the method leading to the answer which should be a full and correct mathematical justification of the given statement. The need for this was highlighted in **Questions 1(i)**, **11(c)**, and **12(i)** in this examination.

Some candidates would also do better if they read the question more carefully and focussed on key words such as 'Hence, …'. The use of this instruction indicates that the previous part or parts of the question should be used. This is often because a particular method is being assessed. Candidates should be aware that when they choose to ignore this key word they may be penalised. This was the case in **Questions 1(ii)**, **9(ii)** and **11(b)(ii)** in this examination.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

(i) Many candidates found this to be a straightforward start to the paper. The most efficient solution was to replace $\cot\theta$ with $\frac{\cos\theta}{\sin\theta}$ and then form the compound fraction $\frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$. Other solutions were acceptable, although those using the minor trigonometric ratios were more likely to produce errors. The majority of candidates worked from left to right. Treating the identity as an equation was not condoned and the few candidates that attempted this were penalised. Occasional slips in notation were made by candidates whose working was poorly presented. Sometimes these slips were recovered. Some candidates combined method steps and lost more marks if they made an error in that step than they may have done if they have shown two steps of working instead of one step. As this question required candidates to 'Show that...' the solution needed to be accurate and complete.

(ii) Again, this part of the question was well answered by very many candidates. Most candidates understood the significance of the word 'Hence...' and used the result from part **(i)** to solve the equation correctly. A small number of candidates included the incorrect value 75.5°, which was penalised as it was within the range required. A few candidates needed to take more care with their rounding to 1 decimal place as the answer 14.4 was not credited.

Answer: **(ii)** 14.5°

Question 2

(a) Many excellent answers were seen to this part of the question.

(b) Candidates were required to complete the Venn diagram with values, rather than algebra, so that the information they were given was accurately shown in the diagram. The most efficient solutions involved no algebra and simply showed the relevant regions completed correctly. This was, in fact, straightforward to do. The candidates who did use algebraic methods often left algebraic expressions in the diagram which was not condoned. A few candidates included all or some of the values 30, 15 and 21 in the Venn diagram. This was penalised as these values should not have been included and these candidates would have improved it they had deleted them once their solution was complete. Some candidates with a completely correct diagram did not interpret n(R) correctly and commonly gave the answer 4. Other candidates did not write the 4 in the diagram, leaving this region empty, and still gave 4 as their answer to n(R). The answer candidates gave for n(R) needed to correspond to their Venn diagram. This was not always the case.

Question 3

Most candidates understood that the correct strategy was to find the value of a and the value of b and then use the cubic expression to find the remainder requested. Most of these candidates presented neat and accurate solutions, earning all 6 marks. Some candidates would have done better if they had read the question a little more carefully as the misread of 15 for –15 was fairly common. Other candidates treated $x - 2$ as a factor, setting p(2) = 0. A few candidates made sign slips when simplifying their equations or solving their simultaneous equations and lost accuracy.

Answer: 60

Question 4

Again, a very good number of candidates found this to be a very accessible question and earned all of the 5 marks available. Most rearranged the linear equation to make y the subject and often substituted this into $9x^2 + 4y^2 = 180$. Some candidates would have done better if they had taken more care with the substitution as slips, such as omitting the square, were seen on occasion. Very many candidates were able to form and solve a correct 3-term quadratic equation which they then correctly factorised. Candidates who formed triple-decker fractions such as $\frac{x^2}{4} + \frac{\left(\frac{6+3x}{2}\right)^2}{9} = 5$ were more likely to make rearrangement errors. A very few candidates omitted the cross terms when squaring their expression or thought that, for example, $3x + 2y = \sqrt{180}$. Candidates who had an incorrect equation were only able to earn the method mark for factorising or solving if a clear method was stated. Most commonly, this was not seen with candidates using their calculator to solve their incorrect equation. This was not credited as the mark was given for the method, rather than the solutions, in this case.

Answer: (2, 6) and (– 4, –3)

Question 5

Candidates needed to interpret each part of this question and decide whether a permutation or combination was appropriate. A few candidates did so correctly. Many candidates either did not use all of the information in the question or were unable to determine whether order was important.

(a) Some candidates understood the context of the question meant that order was important and correctly found the answer using, for example, the most efficient method, ${}^7P_4$. Many candidates did not understand the context and gave the answer 35 from ${}^7C_4$. A few candidates omitted to use the fact that 7 girls were in the drama class and that the 4 girls for the play would be selected from these 7. These candidates thought that the answer was 24 from 4! or ${}^4P_4$. This could not be credited as it significantly eased the solution.

(b) (i) Candidates were more successful in this part with many more earning the mark, although ${}^6P_3 = 120$ was offered by some candidates. The few candidates who were attempting to find the sum of the individual cases often gave the answer 16, usually omitting the possibility of there being 3 Indian singers. This approach was more likely to have an error than using ${}^6C_3$.

(ii) A good number of correct solutions were seen to this part. Most stated a correct product of, for example, combinations and evaluated it correctly. Some candidates formed a sum instead of a product and these candidates may have improved if they had had a better understanding that when combining 1 Chinese singer 'and' 1 Indian singer 'and' 1 British singer, the 'and' indicated that multiplication was the correct operation.

(iii) Some excellent solutions were seen here. However, candidates were slightly less successful in this part, with many multiplying the terms rather than adding them. These candidates may have improved if they had understood that, as the only two cases were 3 Chinese singers 'or' 3 Indian singers, the 'or' indicated that addition was the correct operation. A few candidates gave the answer 15 from ${}^5C_3 + {}^4C_3 + {}^2C_0$, not fully understanding what they were trying to find.

Answers: **(a)** 840 **(b)(i)** 20 **(ii)** 40 **(iii)** 14

Question 6

Some candidates converted the angle 1.5 radians into degrees and used this throughout the question. This was not necessary and candidates were expected to work in radians. Most candidates using degrees gave values that were slightly inaccurate and lost one or both accuracy marks. The majority of candidates were able to earn at least 2 marks in each part.

(i) This was slightly better answered than part **(ii)**. Many candidates earned all 3 marks here. These candidates tended to use the angle in radians and the most efficient methods for finding the arc and chord lengths. Some candidates using the cosine rule to find *DE,* forgot to square root. A few candidates assumed that triangle *OED* was equilateral and used *DE* = 5 cm. Other candidates thought that *EOD* was a right angle and found *ED* using $\sqrt{25 + 25}$. A small number of candidates thought that angle *EOD* was 1.64 radians and did not use the vertically opposite angles property. A few other candidates only added 10 to the sum of the arc and chord lengths.

(ii) Many candidates were able to find the area of the sector correctly and many correctly used the SAS formula, given on page 2 of the examination paper, to find the area of the triangle. Again, those working in radians were more accurate than those who had chosen to convert to degrees. Candidates using less direct methods to find the area of the triangle, such as half base times height, were less likely to be accurate. Some incorrect calculations for the area of the triangle, such as $\frac{1}{2}\times 5\times 5$, were seen.

Answers: **(i)** 34.3 **(ii)** 31.2

Question 7

(i) Most candidates understood that the correct strategy in this part was to find the vector **a** + **c** and then apply Pythagoras' theorem. A few candidates made arithmetic slips when finding **a** + **c** but were able to earn the method mark. Many candidates gave final answers that were inexact decimals. This was not condoned as an exact answer had been requested. Some candidates attempted to find **c** – **a** rather than **a** + **c**, indicating a misunderstanding of the notation. Weaker candidates found |**a**| and |**c**| and summed the results or simply stated |5**i** + 14**j**| as their answer.

(ii) The key to a correct solution in this part was to observe that the scalar of **i** in **a** + *m***b** needed to be 0. A good number of candidates understood this and found the correct value of *m*. Some went on to find the scalar of **j** as 13 which, while correct, was not needed. Other candidates either equated the scalar of **j** to 0 or 1 or equated both scalars to 0 resulting in more than one value of *m*.

(iii) Candidates were slightly more successful in this part as forming and solving one or both of the two equations in *n* should have resulted in a consistent value. Some candidates were able to rearrange the equation in vector form to $n(2\mathbf{i} + 3\mathbf{j}) = 4\mathbf{i} + 6\mathbf{j}$ and write down the answer by inspection. A few candidates went from this step to $n = \frac{4\mathbf{i}+6\mathbf{j}}{2\mathbf{i}+3\mathbf{j}}$ and then stated *n* as the vector 2**i** + 2**j**, rather than a constant.

Answers: **(i)** $\sqrt{221}$ **(ii)** –2 **(iii)** 2

Question 8

(a) Some excellent solutions were seen to this part of the question. The majority of candidates attempted to find a product of the given matrices first and then find the inverse. Some candidates found **AB** instead of **BA**, not recalling that the multiplication of matrices is not commutative. Arithmetic slips were sometimes made when finding the correct product. When finding the inverse matrix from the product, many candidates used the absolute value of the determinant. This is highly likely to be confusion caused by the use of the notation |**A**| for the determinant of **A**. It is strongly suggested that the notation det **A** is used instead, to avoid this confusion. The adjoint (adjugate) matrix was often found correctly. A few candidates attempted to find $\mathbf{A}^{-1}$ first. Some of these tried to combine it with $\mathbf{B}^{-1}$. The majority of these candidates found $\mathbf{B}^{-1}\mathbf{A}^{-1}$, however. A few candidates, having found $\mathbf{A}^{-1}$ then found $\mathbf{BA}^{-1}$, seeming not to have taken care with the brackets in $(\mathbf{BA})^{-1}$.

(b)(i) A good proportion of candidates were able to state the correct order and write it in an acceptable way. The most common incorrect or unacceptable answers were 3 by 2 or (2, 3) or 2 rows and 3 columns.

(ii) Some candidates presented excellent solutions, setting up a general 1 by 2 matrix such as $(a \ \ b)$ to represent **X**. Using this, they formed and solved simple equations. A few candidates gave an answer containing the correct components but that had an incorrect form, such as $\begin{pmatrix} 2 \\ -0.5 \end{pmatrix}$ or (2, –0.5), which was penalised. Some candidates thought that **X** could not be found as $\mathbf{C}^{-1}$ could not be found. Other candidates attempted to find $\mathbf{C}^{-1}$, not understanding that, in order to have an inverse, a matrix needed to be square.

Answers: **(a)** $-\frac{1}{30}\begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$ **(b)(i)** 2×3 **(ii)** $\begin{pmatrix} 2 & -0.5 \end{pmatrix}$

Question 9

Candidates found this question to be quite challenging and it was a good discriminator. Only good candidates were able to earn all 4 marks in part **(i)** and far fewer of these earned full marks in part **(ii)**. Many candidates were using part **(ii)** to try to correct their answer to part **(i)**. Unfortunately, many of these candidates did not have the manipulation skills to understand that their original expression was still incorrect and a final 'correct' line appeared after one or several incorrect lines of working. This was not condoned.

(i) Most candidates understood that the product rule was the appropriate method of finding the derivative. A reasonable number of candidates were able to differentiate $\sqrt{\sin x}$ correctly using the chain rule. Many candidates thought, however, that the derivative was $\frac{1}{2}(\sin x)^{-\frac{1}{2}}$ which earned partial credit or $\frac{1}{2}(\cos x)^{-\frac{1}{2}}$ which was not credited. Some candidates omitted necessary brackets stating, for example, $\frac{1}{2}\sin x^{-\frac{1}{2}}\cos x$, which was not credited unless the brackets were recovered in a later step.

(ii) Full marks could only be given to candidates who had fully correct solutions in part **(i)**. There were a reasonable number of these. Some of these candidates gave very efficient and neatly explained solutions finding the correct answer in two or three steps, demonstrating good understanding. Others of these candidates were able to find the correct answer and, although the working offered was not always completely correct, recovery was allowed. There was some confusion as to whether the integral should be multiplied by 2 or divided by 2. There was also some confusion about how to deal with the extra *x* term in the integral. Again, this was often incorrectly doubled, differentiated or subtracted. Some candidates combined steps and made an error. As no correct step had been seen, accuracy marks could not be awarded in these cases. Candidates offering incorrect solutions to part **(i)** were able to earn a mark in this part for understanding the connection between this part of the question and part **(i)**. A few candidates were able to earn SC1 for stating the correct answer even though their answer to part **(i)** was incorrect, as they had deduced the relationship. Weaker candidates tended to try to integrate each term and expressions such as $\frac{x^2}{2}+\frac{x^5\sin x}{5\times 2(-\cos x)^{\frac{3}{2}}}+8\frac{x^4}{4}(-\cos x)^{\frac{3}{2}}$ were not uncommon.

Answers: **(i)** $4x^3\sqrt{\sin x}+\frac{x^4\cos x}{2\sqrt{\sin x}}$ **(ii)** $2x^4\sqrt{\sin x}+\frac{x^2}{2}+c$

Question 10

(a) (i) Some candidates drew neat and accurate sketches, earning a simple 2 marks. Other candidates did not recognise the function as being quadratic and plotted points, which occasionally impacted on the shape of their curve. Other candidates lost the mark for a correct shape as the maximum was on the *y*-axis or the outer branches had incorrect curvature or were missing. A few candidates had turning points rather than cusps. This was not condoned. Some candidates omitted to record the positions of the roots or incorrectly stated them as –5 and 3. Some weaker candidates, whilst they did not attempt the modulus of the function, had roots in the correct position.

(ii) This question was not well answered. Many candidates stated more than one possible domain and this resulted in an overall domain that was incorrect. Another common incorrect answer offered was $-3 < x < 5$. Candidates offering solutions such as this clearly did not fully understand the requirement that the function needed to be one-one on the domain they suggested. Sometimes the answer given was in incorrect form for a domain with f or *y* being used instead of *x*.

(b) (i) Most candidates stated a correct answer. A few candidates went on to form and solve an equation using their expression and, as the final answer was being marked, were not credited if they did so. A few other candidates found the product of the functions, rather than the composite function, or composed the functions in the incorrect order.

(ii) Again, a very good number of correct answers were seen. Occasionally arithmetic or sign slips were seen, resulting in the loss of the accuracy mark. Sometimes candidates made method errors, such as incorrectly factorising, and lost the method mark. A few candidates found a correct expression for the inverse function and then multiplied it by *x*, misunderstanding the function notation. Candidates finding both g^{-1} and h^{-1} usually found $h^{-1}g^{-1}$ or, occasionally, hg^{-1}. A few candidates forgot whether they were trying to make *x* or *y* the subject and gave a circular argument, restating the original function as the inverse function. A few candidates offered a final answer in terms of *y*. This was not accepted.

(c) Some good candidates could see the correct answer and simply stated *a*. Other candidates misinterpreted the inverse function notation and gave answers of $-a$ or $\frac{1}{a}$ or unsimplified forms such as $pp^{-1}(a)$. Occasionally, incorrect answers in terms of *b* were stated or no response was offered.

Answers: **(b)(i)** $\frac{4}{3x-1}$ **(ii)** $(hg)^{-1}(x) = \frac{4+x}{3x}$ **(c)** a

Question 11

(a) A good proportion of candidates gave a correct answer to this part. Most candidates were able to rewrite the expression with the correct power. Many of these increased the power by 1 and divided by the new power. A few errors were made when dealing with the 2. Some candidates multiplied instead of dividing the expression by 2 and others completely omitted it. Candidates who clearly showed the $\frac{4}{3}$ and the 2 but mis-combined them could still earn a mark. Candidates who combined two steps incorrectly and only stated, for example, $\frac{3}{2}(2x-1)^{\frac{4}{3}}$ may simply have multiplied by 2 instead of dividing, but as there was no clear evidence of that, they could not be credited. A few candidates differentiated instead of integrating. A small number of candidates incorrectly wrote the power as $\frac{3}{2}$ or 3. At this stage of the paper, it was expected that candidates would be able to write the cube root correctly as a power. Very weak candidates attempted to cube root $2x - 1$ term by term.

(b) (i) Again, some good answers were seen here, with many candidates earning at least one mark and many earning both marks. Convincing attempts to integrate, such as $-4\cos 4x$ or $-\cos 4x$, which earned a mark for a negative multiplier of $\cos 4x$, or $\frac{\cos 4x}{4}$ were fairly frequent. Weak candidates offered $-4\cos x$ or incorrectly raised the power of the argument of the cosine.

(ii) The method mark in this part was dependent on a convincing attempt to integrate in part **(b)(i)**. In order to demonstrate that they were using part **(b)(i)**, as required by the key word 'Hence...', and not simply using their calculator, candidates needed to show the correct substitution of the limits into their expression. Many candidates did this and earned the method mark. Those who ensured that their calculator was in the correct mode and entered their expressions carefully usually earned the second mark also. Other candidates would have improved if they had checked that their calculator was in radian mode and if they had taken a little more care with the negative and minus signs in this part of the question as a few candidates omitted to write down one of the middle signs resulting in an incorrect method.

(c) In this part, candidates needed to show full method in order to earn full marks, since the answer was given. A reasonable number of candidates did demonstrate fully that the answer was 3. This included evidence as to why $e^{\frac{\ln 8}{3}}$ was equal to 2. Many candidates earned 4 marks, having omitted this key part of the solution. A few candidates omitted the 6 – 3 from the last step of which resulted in an incomplete argument and therefore they lost the final mark. Some candidates did not show the substitution of the limits into the integrated expression. This resulted in a significant loss of marks. A few candidates multiplied by $\frac{1}{3}$ instead of multiplying by 3 or simply thought that the integral of $e^{\frac{x}{3}}$ was $e^{\frac{x}{3}}$. These candidates were given the benefit of the doubt and allowed some credit on this occasion. Weak candidates tended to increase the power by 1 and divide by the new power or offered $\frac{3}{x}e^{\frac{x}{3}}$. These responses were inappropriate and not credited.

Answers: **(a)** $\frac{3}{8}(2x-1)^{\frac{4}{3}}+c$ **(b)(i)** $-\frac{\cos 4x}{4}+c$ **(ii)** $\frac{1}{4}$

Question 12

(i) The most efficient method of finding a correct expression for *r* in terms of *h* was to use basic trigonometry and the tangent of the angle $\frac{\pi}{12}$ radians. Candidates were given exact trigonometric values at the start of the question to help with this and a good number were successful. Some candidates thought *h* was the slant height and used the sine of the angle. A few candidates successfully used the sine rule. The cosine rule was slightly more complicated and resulted in more errors, when attempted. Many candidates found a correct expression and, if they were sufficiently careful with their substitution, were able to earn all 4 marks. The formula for the volume had been given at the start of the question. In order to earn the mark for substitution, the substitution needed to be done correctly. Some candidates made this unnecessarily complex and omitted brackets that were necessary at the substitution stage in order that their expression be correct. Some candidates would have improved if they had shown at least three terms in the expansion of $(2-\sqrt{3})^2$. As the answer had been given, this was a requirement for the final mark to be awarded. Some candidates were unable to use the information given effectively. These candidates tended to find their expression using the formula for *V* and then substitute back into *V*, creating a circular argument.

(ii) Many candidates were able to find the correct rate here. Some candidates were better at using the correct mathematical notation than others. On this occasion, incorrect use of notation was condoned if a correct method was otherwise offered. Candidates who were unable to form a correct numerical chain rule expression for $\frac{dh}{dt}$ sometimes earned the mark for making a correct general chain rule statement with correct variables. Other candidates were able to earn a mark for differentiating *V* in terms of *h* correctly. Some candidates did make this more complex than necessary by applying the quotient rule to the expression for *V* and often made a slip when differentiating 3. Some candidates had all the correct components needed to find the answer but multiplied 30 by $\frac{dV}{dh}$ when $h = 5$, rather than dividing. A few alternative methods were offered and most of these involved clear and correct use of calculus, as required. A very small number of candidates gave an exact answer. This was not condoned as a decimal had been requested and this was a question in context, meaning the exact answer was not appropriate. Occasionally, candidates made premature approximation errors, resulting in a loss of accuracy. Very weak candidates tended to evaluate *V* when $h = 5$.

Answers: **(i)** $r=(2-\sqrt{3})h$ **(ii)** 5.32